DOCUMENT RESUME

ED 235 220 TM 830 640

AUTHOR Samejima, Fumiko
TITLE A Latent Trait Model for Differential Strategies in Cognitive Processes.
INSTITUTION Tennessee Univ., Knoxville. Dept. of Psychology.
SPONS AGENCY Office of Naval Research, Arlington, Va. Personnel and Training Research Programs Office.
REPORT NO ONR/RR-83-1
PUB DATE Jun 83
CONTRACT N00014-81-C-0569
NOTE 65p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Classification; *Cognitive Processes; Estimation (Mathematics); Individual Psychology; *Latent Trait Theory; Mathematical Models; *Maximum Likelihood Statistics
IDENTIFIERS *Tailored Testing

ABSTRACT

Some cognitive psychologists, who have tried to approach psychometric theories, say that the psychometric approach does not provide them with theories and methods with which they can deal with differential strategies. In this paper, a general latent trait model for differential strategies in cognitive processes is proposed which includes three different response levels: the nominal response level, the graded response level, and the dichotomous response level. The general graded response model is further classified into the homogeneous and the heterogeneous cases. Also the maximum likelihood estimation of the individual's latent trait is discussed. (Author/PN)

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ED235220

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A LATENT TRAIT MODEL FOR DIFFERENTIAL STRATEGIES IN COGNITIVE PROCESSES

FUMIKO SAMEJIMA

UNIVERSITY OF TENNESSEE

KNOXVILLE, TENN. 37996-0900

JUNE, 1983

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R01-1068-71-002-84

TM 830650

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ONR/Research Report 83-1	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Latent Trait Model for Differential Strategies in Cognitive Processes		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Dr. Fumiko Samejima		8. CONTRACT OR GRANT NUMBER(s) N00014-81-C-0569
9. PERFORMING ORGANIZATION NAME AND ADDRESS Personnel and Training Research Programs Office of Naval Research Arlington, VA 22217		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS PE: 61153N; PROJ:RR-042-04 TA: RR 042-04-01 WU: NR 150-467
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Operating Characteristic Estimation
Tailored Testing
Latent Trait Theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

(Please see reverse side)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

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A LATENT TRAIT MODEL FOR DIFFERENTIAL STRATEGIES IN COGNITIVE PROCESSES

ABSTRACT

Some cognitive psychologists, who have tried to approach psychometric theories, say that they do not provide them with theories and methods with which they can deal with differential strategies. They are not exactly right. As early as in the late nineteen-sixties, the heterogeneous case of the graded response level in the context of latent trait theory was proposed (A general model for the operating characteristic of graded item response, Psychometric Laboratory Report No. 55, 1967, University of North Carolina) as a model for cognitive processes. Some useful hints for differential strategies are seen elsewhere (A general model for free-response data, Psychometrika Monograph No. 18, 1972, Section 3.4) under the title, "Multi-correct and multi-incorrect responses."

In the present paper, following the same line, a general latent trait model for differential strategies in cognitive processes is proposed, and the maximum likelihood estimation of the individual's latent trait is discussed.

The research was conducted at the principal investigator's laboratory, 405 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked for her as assistants include Paul S. Changas, Vicki Newton, Donald Reece Danna, Cornelia Chapman, Hossein H. Kord, Donna Reynolds, Nancy Cossentine, and Philip Livingston.

TABLE OF CONTENTS

		Page
I	Introduction	1
II	Cognitive Process as Digraphs	3
III	Differential Strategy Tree	9
IV	A General Model for Differential Strategies	13
V	Homogeneous Case	23
VI	Single Strategy Case	27
VII	Discussion and Conclusions	37
	References	39
	Appendix	43

I Introduction

A general model for free-response data has been proposed by the author (Samejima, 1972) which includes three different response levels, i.e., the nominal response level, the graded response level, and the dichotomous response level. The general graded response model is further classified into the homogeneous and the heterogeneous cases.

Two of the most common uses of the graded response model may be for test items which have more than two item score categories, e.g., A, B, C, D and F, and for questionnaire items with multiple response categories, e.g., "strongly disagree," "disagree," "agree," and "strongly agree." The rationale behind this general model is, however, originally based upon the general sequential cognitive process, which has been described elsewhere (Samejima, 1967).

Let θ be the latent trait underlying the cognitive process. When the latent space is unidimensional, θ is the set of all real numbers, and, when it is multidimensional, θ is a vector of k sets of real numbers, such that

$$\theta = (\theta_1, \theta_2, \cdots, \theta_k)' \quad , \tag{1.1}$$

with k indicating the dimensionality. In the present paper, however, we deal solely with the unidimensional latent space.

Let us take problem solving as an example. Suppose that for solving the problem g we need m_g sequential subprocesses. Let y_g denote the attainment category or attainment score. One must

successfully follow all the m_g sequential subprocesses in order to solve the problem g, so the attainment category y_g assumes integers, 0 through (m_g+1), with $y_g = 0$ indicating that the individual subject has successfully followed no subprocesses, and with $y_g = m_g$ meaning that he has completed all m_g subprocesses required to solve the problem. The additional attainment score, (m_g+1), represents that the subject has successfully followed the additional subprocess which is hypothesized at the end of the entire sequence of subprocesses. Since no one can accomplish this, the conditional probability, given θ, with which the subject obtained the attainment score y_g equals zero, regardless of the fixed value of θ. With this setting, we can see that the general graded response model can readily be applied to the single strategy case of problem solving. In this paper, however, our main objective is to approach a general model for the multiple strategy case, or differential strategies, in the context of latent trait theory.

The operating characteristic of a discrete response relates the response to the latent trait θ. In the context of problem solving, this operating characteristic is considered with respect to an attainment score, or to a union of attainment scores. It takes an important role in estimating the subject's latent trait from his performance.

As concrete mathematical models, we shall consider those in the homogeneous case, which was expanded and generalized from the one on the graded response level.

II Cognitive Process as Digraphs

It is a fairly common phenomenon that there exist more than one way of solving a problem. In proving mathematical theorems, for example, we often find one proof plus several alternative proofs for one theorem. To represent those differential strategies graphically, Figure 2-1 presents a simple example of a two strategy case in the form of a graph, each strategy having a small number of subprocesses. In this example, if we take the first strategy, then we must traverse the path, $v_0\ v_1\ v_2\ v_3\ v_4$, whereas we must follow another path $u_0\ u_1\ u_2\ u_3\ u_4\ u_5$ if we take the second strategy, in order to solve the problem. Note that, although we have duplicate symbols, $v_0 = u_0$, $v_1 = u_1$, $v_3 = u_4$ and $v_4 = u_5$. These two paths represent the two successful strategies for solving our hypothetical problem, and are shown as two directed subgraphs of the original graph, or digraphs, in Figure 2-2.

When the subject falters, he may not follow either of the two paths shown in Figure 2-2. In such a case, we need additional arcs in the digraph presented as Figure 2-1, examples of which are shown by thin lines in Figure 2-3. Two examples of the directed subgraphs of Figure 2-3, which represent "faltering," are presented in Figure 2-4. In the first digraph, the subject initially took the first strategy, but came back from the vertex v_2 to the vertex v_1, and then took the second strategy until he solved the problem. In the second digraph, the subject initially took the second strategy up to the vertex u_3, then came back to the vertex u_1 and took the first strategy. These are rather simple examples adding one cycle to each path in Figure 2-2, making the strategy a trail instead of a

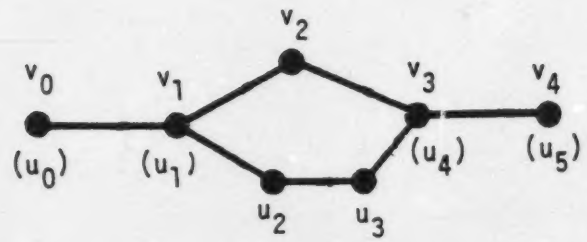

FIGURE 2-1

Example of Two Strategy Case Drawn as a Graph.

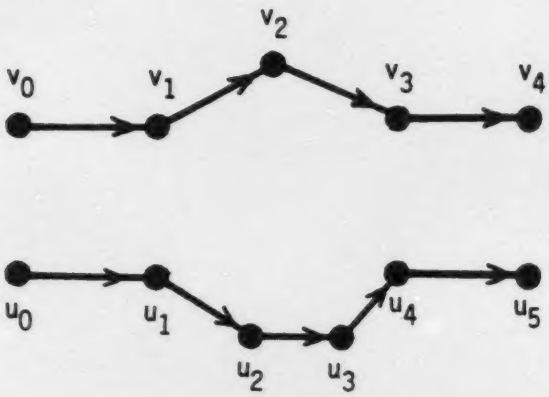

FIGURE 2-2

Two Directed Subgraphs Representing the Two Strategies.

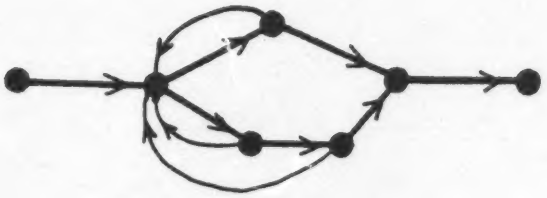

FIGURE 2-3

Digraph Including Two Loops and a Cycle Representing the Subject's Faltering.

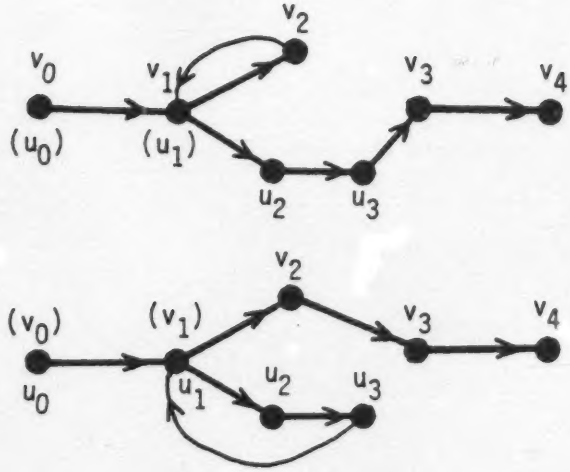

FIGURE 2-4

Two Examples of Directed Subgraphs Representing the Subject's Faltering.

path. We can conceive of more complex examples, however, in which the subject traverses several cycles repeatedly in a single walk, for instance.

So far in our graphs all the vertices are those included in either the first or the second, or both, of the two directed subgraphs of Figure 2-2, which represent the two successful strategies for solving the problem. In our cognitive process, however, we often choose wrong strategies which do not lead to the solution of the problem. Figure 2-5 illustrates such situations, in which hollow circles are added to the solid circles of Figure 2-1, and dashed arcs are added to imply additional paths, which represent wrong strategies. Two examples of such unsuccessful strategies are illustrated in Figure 2-6. In the first subgraph, the subject followed the second successful strategy up to the vertex u_2 , and then branched himself to a subsequence of two subprocesses which led him to a "dead end." The second subgraph illustrates a similar case in which the subject followed the second successful strategy up to the vertex u_3 , and then followed a "dead end" subsequence of two subprocesses.

If the subject took a wrong strategy and found out he could not solve the problem if he sticks to that strategy, he may give up, or he may come back to a previous point in the path and try another strategy. To illustrate this latter situation, additional arcs are necessary in the graph of Figure 2-5. In Figure 2-7, some dotted arcs are added to imply new cycles which the subject may follow when he "comes back." Two examples of such trails are given in Figure 2-8. In the first example, the trail indicates that the subject branched himself to a wrong strategy after he reached the vertex u_2 of the second successful strategy,

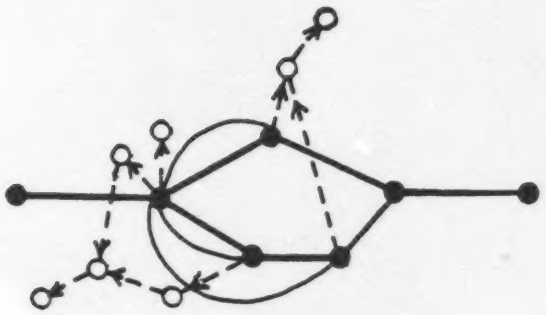

FIGURE 2-5

Digraph Added by Unsuccessful Strategies
(Dashed Line).

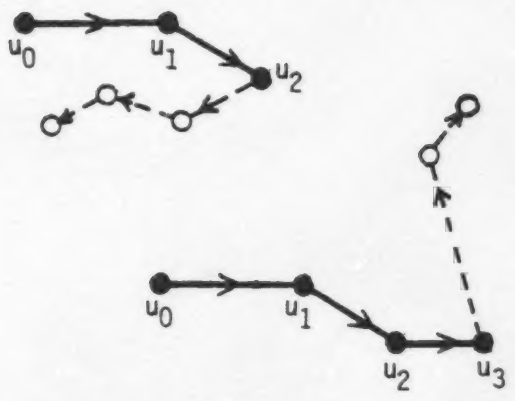

FIGURE 2-6

Two Examples of Unsuccessful Strategies.

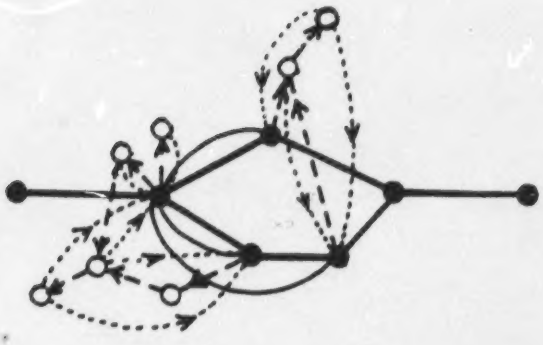

FIGURE 2-7

Digraph Added by Loops and Cycles Representing the Subject's Faltering after He Has Taken Unsuccessful Strategies.

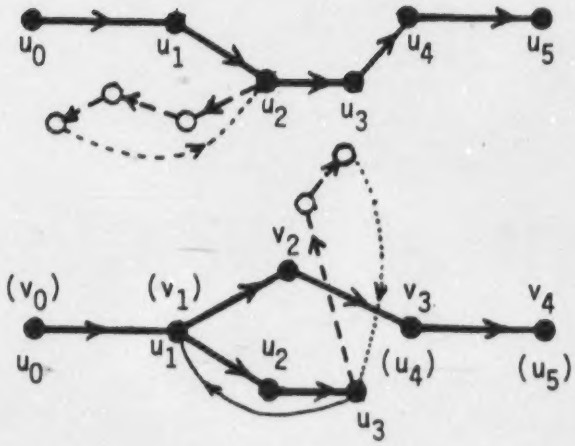

FIGURE 2-8

Two Trails Representing the Subject's Faltering when He Has Taken Unsuccessful Strategies.

followed the path until he reached the "dead end," and then came back to u_2 and followed the rest of the second successful strategy. In the second example, the subject followed up to the vertex u_3 of the second successful strategy, branched himself to a wrong strategy, followed the path to the "dead end," and came back to u_3, went back to the vertex u_1 $(= v_1)$, and then took the first successful strategy until he solved the problem.

We have seen just a small number of examples of conceptualizing the cognitive processes involved in problem solving, and there are a great many other varieties of paths, trails, and walks which might represent a particular subject's cognitive process. The subject may walk the same cycles over and over again, for example, or he may stop at, say, the vertex v_2 in the path representing the first successful strategy and then not proceed, and so forth. In any case, graphic representation illustrated in this section will benefit us in systematizing those complex situations.

III Differential Strategy Tree

As was referred to in the first section, latent trait theory relates the subject's behavior to his position in the latent space. In problem solving, his behavior is observed in terms of the degree of attainment toward the solution of the problem.

It is obvious that following those cycles presented in Figures 2-3 and 2-7 itself will not directly improve the subject's degree of attainment toward the solution of the problem. For this reason, as far as

we consider the degree of attainment toward the solution, we can more or less ignore the subject's traversing on cycles. In other words, the things that count are the paths in those graphs, rather than trails or walks which may include one or more cycles. Thus the graph of Figure 2-7 may be reduced to the one presented in Figure 3-1. This implies that, when we consider the subject's degree of attainment toward the solution, the first trails in Figures 2-4 and 2-8 are treated as equivalent to the second path in Figure 2-2, and the second trails in those two figures are treated as equivalent to the first path in Figure 2-2.

We also notice that even the same subject in exactly the same problem solving situation may take different strategies, if it is given repeatedly. Thus even if his latent trait and the given situation stay the same, he may have a higher probability of solving the problem when he chooses one strategy in preference to the others. In other words, there is no reason to assume that for a given individual of trait θ the probability of success stays the same when he chooses different successful strategies. In the illustration presented in the preceding section, the probability with which the subject of trait θ solves the problem using the first successful strategy may not be the same as the one when he uses the second successful strategy, even though the edge $(v_3\ v_4)$ is the same as the edge $(u_4\ u_5)$.

The above observation suggests that, as far as we consider the degree of attainment and its corresponding operating characteristic, it may be more convenient to represent the set of all strategies as a <u>tree</u>. Figure 3-2 presents such a tree obtained from the graph in Figure 3-1 for

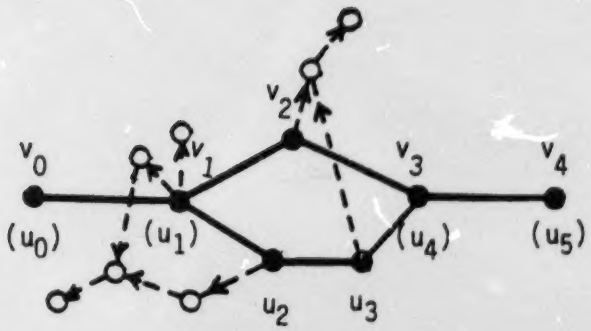

FIGURE 3-1

Digaraph Excluding All Cycles.

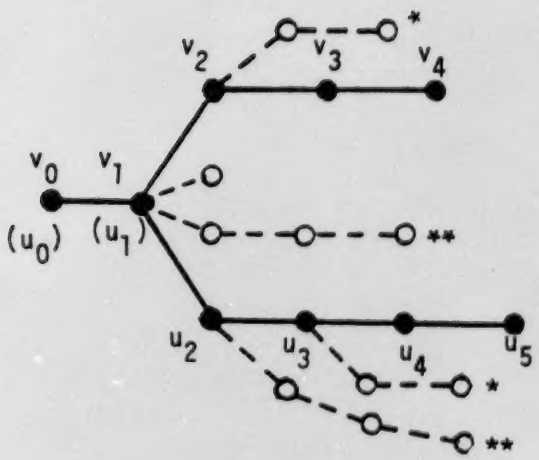

FIGURE 3-2

Differential Strategy Tree Including Both Successful and Unsuccessful Strategies.

the example discussed in the preceding section. Note that not only are the two points v_4 and u_5 in Figure 3-2 the same point in the original graph, Figure 3-1, but also the two hollow circles marked with * in Figure 3-2 are a single point in Figure 3-1, and so are the two marked with ** . Hereafter, we shall call this kind of tree the differential strategy tree. It is a kind of directed graph which contains several paths representing different strategies, joining a common initial endpoint with the other distinct endpoints. We may call this initial point a nothing point, indicating that, "nothing has been accomplished yet." We may call the other endpoints for the successful strategies solution points, meaning, that the "solution has been reached." Since no one can surpass a solution point, it also represents a hypothesized attainment score which no one can obtain.

In theory, we should be able to estimate the operating characteristic for each of the subprocesses in Figure 3-2, which are represented by solid or dashed edges, if appropriate models are discovered. In practice, however, the development of such a model, or models, and its validation will involve much more complicated problems, if, at the initial stage, we include unsuccessful strategies in addition to the successful strategies. For this reason, here we shall consider models for successful strategies only, and solely use the information related to the successful strategies. When some model thus constructed has been validated, we can proceed to explore the operating characteristics of the various subprocesses of unsuccessful strategies, in a way similar to that of estimating the plausibility function of each distractor of the multiple-choice test item

after we have estimated the operating characteristic of the correct answer (cf. Samejima, ONR Final Report, 1981). Thus the path representing the first unsuccessful strategy in Figure 3-2 is treated here as equivalent to the path $v_0\ v_1\ v_2$ in the first successful strategy, those representing the second and third unsuccessful strategies to the path $v_0\ v_1$ in the first successful strategy, the path for the fourth unsuccessful strategy to the path $u_0\ u_1\ u_2\ u_3$ in the second successful strategy, and the one for the fifth to the path $u_0\ u_1\ u_2$ in the second successful strategy.

The example we have observed is simple and easy. In reality, however, we should expect more complicated differential strategy trees. Figures 3-3 and 3-4 present another, a little more complicated example of the digraph and corresponding differential strategy tree, in which only successful strategies are drawn. Thus in our second example, we have five successful strategies and five solution points.

IV A General Model for Differential Strategies

Let w denote the number of successful strategies for solving the problem g. This number equals the number of solution points in the differential strategy tree, which was illustrated in the preceding section. Each of those w strategies consists of m_{gi} $(i = 1,2,\ldots,w)$ subprocesses, and they are represented by the vertices, excluding the first and last, in the digraphs and in the differential strategy trees presented in the previous two sections. In the example which was first presented and discussed in Section 2, $w = 2$, $m_{g1} = 3$ and $m_{g2} = 4$, and the subprocesses are represented by four edges, $(v_0\ v_1)$, $(v_1\ v_2)$, $(v_2\ v_3)$

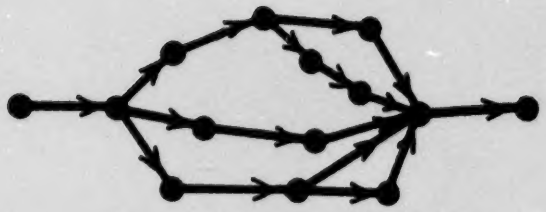

FIGURE 3-3

Another Example of Digraph Representing
Five Successful Strategies.

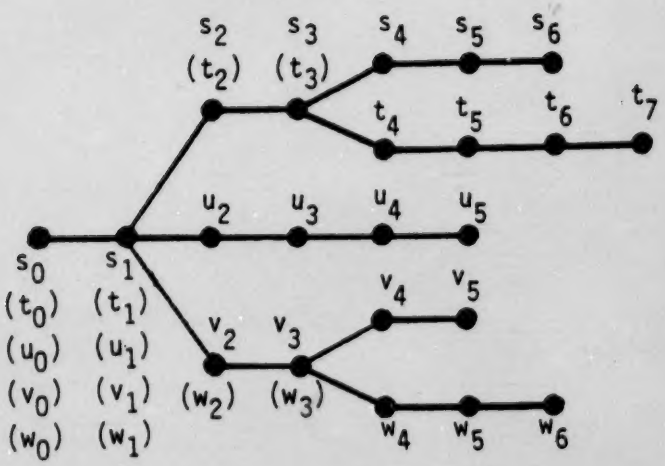

FIGURE 3-4

Differential Strategy Tree of the Second
Example Including Successful Strategies
Only.

and $(v_3\ v_4)$, in the path $v_0\ v_1\ v_2\ v_3\ v_4$ representing the first successful strategy, and by five edges, $(u_0\ u_1)$, $(u_1\ u_2)$, $(u_2\ u_3)$, $(u_3\ u_4)$ and $(u_4\ u_5)$, in the path $u_0\ u_1\ u_2\ u_3\ u_4\ u_5$ which represents the second successful strategy. Hereafter, we shall call this example Example 1. It is fairly common that in the original digraph some subprocesses are shared by two or more different strategies, as we can see in Example 1, i.e., $(v_0\ v_1) = (u_0\ u_1)$ and $(v_3\ v_4) = (u_4\ u_5)$.

Let y_{gi} be the attainment score indicating the degree of attainment of the subject's performance toward the solution of the problem g, which takes on integers 0 through m_{gi} when the subject chooses the strategy i. Figure 4-1 presents several attainment scores assigned to trails which were taken from Example 1. Also Figure 4-2 shows the attainment scores assigned to each edge of the differential strategy tree of the other example, which was presented at the end of Section 3. Hereafter, we shall call this other example Example 2.

A general model for differential strategies concerns the assignment of an operating characteristic to each attainment score y_{gi} of each of the w strategies i for solving the problem g. By such an operating characteristic we mean the conditional probability with which the subject of trait θ chooses the strategy i and obtains the attainment score y_{gi}. We notice, however, that in general, if the subject's performance stopped before branching, there is no way to decide which of the two or more strategies he would have taken. This fact is well represented in the differential strategy tree. For example, in Figure 3-4, $(s_0\ s_1)$, $(t_0\ t_1)$, $(u_0\ u_1)$, $(v_0\ v_1)$ and $(w_0\ w_1)$ are a single edge before

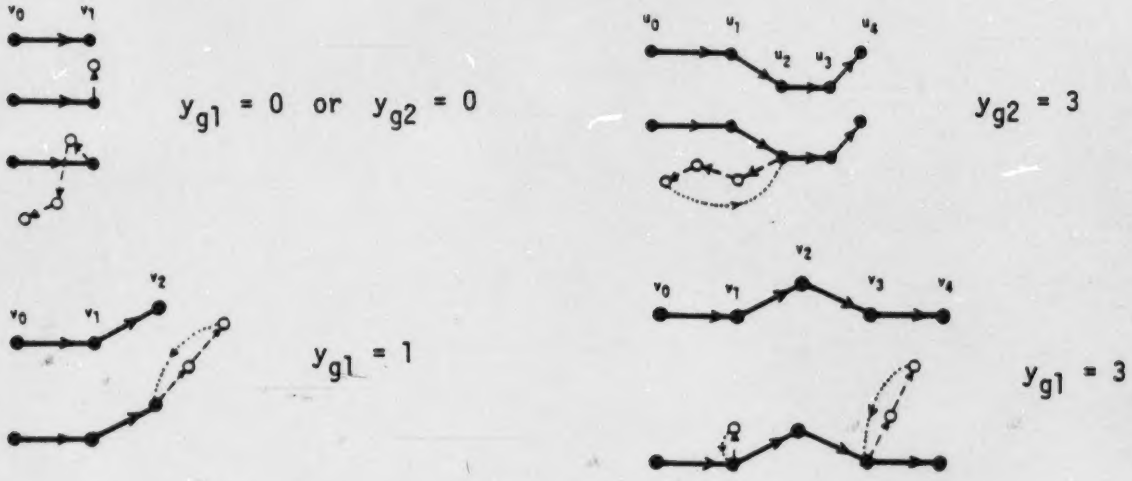

FIGURE 4-1

Examples of Attainment Scores Given to Different Subgraphs in Example 1.

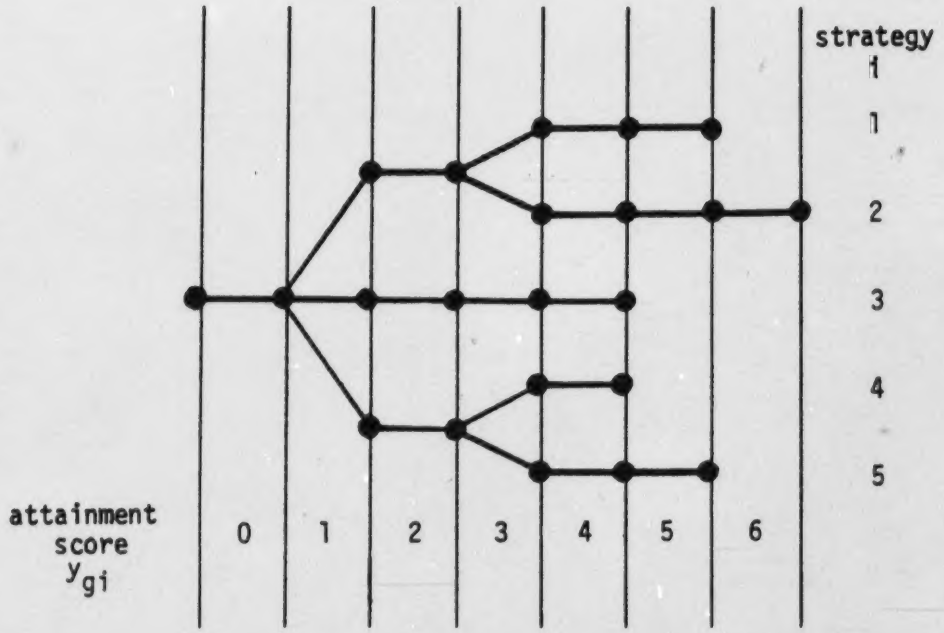

FIGURE 4-2

Attainment Scores Assigned to Each Line of the Differential Strategy Tree in Example 2.

the first branching, and as long as the subject's performance stays with this subprocess he could be heading for any one of the five successful strategies. We can also say the same for the edge which is both $(s_1\ s_2)$ and $(t_1\ t_2)$, for the one which is both $(v_1\ v_2)$ and $(w_1\ w_2)$, and so forth. Thus we must assign a single operating characteristic for each edge of the differential strategy tree. Since each edge represents a union of one or more attainment scores, the operating characteristic is to be assigned to each union. For instance, following an appropriate model, a single operating characteristic will be assigned to the union of $y_{gi} = 0$ for $i = 1,2,3,4,5$, and the same model will provide us with an operating characteristic solely for $y_{g4} = 3$, and so on. For convenience, we shall choose the smallest i in each union, and let y^*_{gsi} denote such a union with s for the actual attainment score. In Example 2, for instance, $y^*_{g01} = (y_{g1}=0) \cup (y_{g2}=0) \cup (y_{g3}=0) \cup (y_{g4}=0) \cup (y_{g5}=0)$, and $y^*_{g34} = (y_{g4}=3)$, and none of the four unions, y^*_{g02}, y^*_{g03}, y^*_{g04} and y^*_{g05}, exists.

Let $M_{y^*_{gsi}}(\theta)$ denote the conditional probability with which the subject of trait θ obtains s as his attainment score in one of the strategies which belongs to y^*_{gsi}, with the joint condition that he has already obtained the score $s-1$. Since there is no preceding attainment score for $y_{gi} = 0$, and y^*_{g01} is the union of $y_{gi} = 0$ for all the w strategies, the attainment function $M_{y^*_{g01}}(\theta)$ takes on unity throughout the whole range of θ. On the other hand, since $y_{gi} = m_{gi}+1$ is a hypothesized attainment score which is higher than the full score m_{gi}, the attainment function $M_{y^*_{g(m_{gi}+1)i}}(\theta)$ assumes zero for the entire range of θ for each of the w strategies. Thus we can write

$$(4.1) \qquad M_{y^*_{gsi}}(\theta) \begin{cases} = 1 & i = 1, \quad s = 0 \\ = 0 & i = 1,2,\ldots,w, \quad s = m_{gi}+1 . \end{cases}$$

Note that in (4.1) the first line indicates a single function for the union of $y_{gi} = 0$ for $i = 1,2,\ldots,w$, while the second line indicates w separate functions for $i = 1,2,\ldots,w$.

Hereafter, we shall assume that each attainment function $M_{y^*_{gsi}}(\theta)$ is three-times differentiable with respect to θ. Note that this assumption does not contradict (4.1).

Let $P^*_{y^*_{gsi}}(\theta)$ be the conditional probability assigned to the union of attainment scores y^*_{gsi}, with which the subject of trait θ chooses a strategy which belongs to y^*_{gsi} and obtains the attainment score s or greater. We shall call this function the <u>cumulative operating characteristic of the attainment score union</u> y^*_{gsi}.

From the definitions of this function and the attainment function $M_{y^*_{gsi}}(\theta)$, we can write

$$(4.2) \qquad P^*_{y^*_{gsi}}(\theta) = \prod_{\substack{s=0 \\ *i}}^{s} M_{y^*_{gk(*i)}}(\theta) ,$$

where $*i$ indicates the closest integer less than or equal to i for which the union of attainment scores exists. In particular, we have

$$(4.3) \qquad P^*_{y^*_{gsi}}(\theta) \begin{cases} = 1 & s = 0 \\ = M_{y^*_{gsi}}(\theta) & s = 1 \\ = 0 & s = m_{gi}+1 . \end{cases}$$

Note that the first line of (4.3) indicates a single function for the

union of $y_{gi} = 0$ for $i = 1,2,\ldots,w$, the second line indicates one or more functions depending upon the branching, and the third line indicates the w separate functions for $i = 1,2,\ldots,w$. In Example 1, the second line includes two functions, while in Example 2 it includes three functions.

Let $A^{*}_{y^{*}_{gsi}}(\theta)$ be the first derivative or the natural logarithm of the cumulative operating characteristic $P^{*}_{y^{*}_{gsi}}(\theta)$, that is,

$$A^{*}_{y^{*}_{gsi}}(\theta) = \frac{\partial}{\partial\theta} \log P^{*}_{y^{*}_{gsi}}(\theta) \quad . \tag{4.4}$$

Note that this function is unchanged when $P^{*}_{y^{*}_{gsi}}(\theta)$ is multiplied by a constant. To be more specific, let $\Psi(\theta)$ be a function defined by

$$P^{*}_{y^{*}_{gsi}}(\theta) = c_1 + (c_2 - c_1)\,\Psi(\theta) \quad , \tag{4.5}$$

where $0 \leq c_1 < c_2 \leq 1$. The first derivative of the natural logarithm of $P^{*}_{y^{*}_{gsi}}(\theta)$ is given by $\frac{\partial}{\partial\theta}\log[c_1 + (c_2 - c_1)\Psi(\theta)]$, which equals $\frac{\partial}{\partial\theta}\log\Psi(\theta)$ if, and only if, $c_1 = 0$. The formula (4.5) has been observed in a somewhat different context (Samejima, ONR/RR-82-1) where $P^{*}_{y^{*}_{gsi}}(\theta)$ is replaced by any strictly increasing function of θ with zero and unity as its two asymptotes. We have called the four different types of functions derived from (4.5) Types A, B, C and D, depending upon the values of c_1 and c_2, i.e., the function is of Type A when $0 = c_1 < c_2 = 1$; of Type B when $0 < c_1 < c_2 = 1$; of Type C when $0 = c_1 < c_2 < 1$; and of Type D when $0 < c_1 < c_2 < 1$, respectively. This implies that the cumulative operating characteristic of Types A and C may share the same function for

$A^*_{y^*_{gsi}}(\theta)$, and we can say the same for those of Types B and D. The necessary and sufficient condition that $M_{y^*_{gsi}}(\theta)$ be strictly increasing in θ is that the inequality

$$(4.6) \qquad A^*_{y^*_{gsi}}(\theta) > A^*_{(y^*_{g(s-1)(*i)})}(\theta) \qquad s = 1,2,\ldots,(m_{gi}+1)$$

holds almost everywhere with respect to θ .

The operating characteristic, $P_{y^*_{gsi}}(\theta)$, defined for the union of the attainment scores Y^*_{gsi} is given by

$$(4.7) \qquad P_{y^*_{gsi}}(\theta) = P^*_{y^*_{gsi}}(\theta) - \sum_{j^*} P^*_{y^*_{g(s+1)j}}(\theta) \ ,$$

where $\sum_{j^*}$ indicates the summation over all the strategies j branching from the point which lies immediately after the line representing y^*_{gsi} .

This operating characteristic can be considered as the likelihood function in estimating the subject's latent trait θ . If there are more than one problem satisfying the conditional independence of the attainment scores across the different strategies, given θ , the maximum likelihood estimation of the subject's latent trait can be performed on the basis of the response pattern V , such that

$$(4.8) \qquad V = (y_{1i_1}, y_{2i_2}, \cdots, y_{gi_g}, \cdots, y_{ni_n})'$$

for the n problem solving tasks, where i_g is a strategy for solving the problem g and y_{gi_g} is the attainment score when the subject chooses

the strategy i_g for solving the problem g. Let $P_V(\theta)$ be the operating characteristic of the specific response pattern V. We can write

$$(4.9) \qquad P_V(\theta) = \prod_{*} P_{y^*_{gsi}}(\theta) \ ,$$

where $\prod_{*}$ indicates the multiplication over every union y^*_{gsi} to which an element of V belongs.

It is beneficial to search for a family of models which provide us with a unique maximum for every possible response pattern given by (4.8). This can be done as a generalization of the unique maximum condition proposed for the graded response model (cf. Samejima, 1969, 1972). Two of such models will be proposed and observed in Section 5.

The <u>basic function</u>, $A_{y^*_{gsi}}(\theta)$, for the union of attainment scores y^*_{gsi} is defined by

$$(4.10) \qquad A_{y^*_{gsi}}(\theta) = \frac{\partial}{\partial\theta} \log P_{y^*_{gsi}}(\theta) \ .$$

The maximum likelihood estimate, $\hat{\theta}_V$, of the subject's latent trait based upon his response pattern is given as the solution of the likelihood equation such that

$$(4.11) \qquad \frac{\partial}{\partial\theta} \log P_V(\theta) = \sum_{*} \frac{\partial}{\partial\theta} \log P_{y^*_{gsi}}(\theta)$$

$$= \sum_{*} A_{y^*_{gsi}}(\theta) \ ,$$

where $\sum_{*}$ indicates the summation over every union y^*_{gsi} to which an

element of V belongs. A sufficient condition that a unique modal point exists for the likelihood function $P_V(\theta)$ of each and every response pattern V is that this basic function is strictly decreasing in θ with non-negative and non-positive values as its two asymptotes, for every union y^*_{gsi}. This can be shown in the same way that we did for the basic function $A_{x_g}(\theta)$ of the graded item score x_g (cf. Samejima, 1969). For brevity, sometimes we call this condition the unique maximum condition.

Multi-correct and multi-incorrect responses have been discussed briefly (Samejima, 1972, Sections 3.4 and 4.2), and it has been pointed out that all those response categories can be syndrome categories, i.e., response categories whose operating characteristics satisfy the unique maximum condition (Samejima, 1969, 1972). Those cases are included in Situation B (Samejima, 1972, Section 3.2), where there exist more than one response category whose operating charateristic is of Type (i) (or of Type (ii)), and, at least, one response category whose operating characteristic is of Type (ii) (or of Type (i)), and, in addition there may be one or more response categories whose operating characteristics are of Type (iii). It should be recalled that by Types (i), (ii) and (iii) we mean strictly increasing, strictly decreasing, and unimodal operating characteristics which satisfy the unique maximum condition. In Situation B, the sum total of the upper asymptotes of all the operating characteristics of Type (i) should equal unity. This implies that, as θ tends to positive infinity, the operating characteristic of each correct answer approaches a finite positive value less than unity. We notice that such operating characteristics belong to the Type C model (Samejima, ONR/RR-82-1).

Similarities between the differential strategies in problem solving and the multi-correct responses in testing are obvious. If we consider two or more different strategies which lead to the solution of the problem as two or more different answers to a question, then they will be treated as multi-correct responses. We can see that the concept of multi-correct responses can be transferred to differential strategies, when there exist more than one successful strategy in solving a problem.

V Homogeneous Case

The Homogeneous case of the graded response level has been developed and discussed (Samejima, 1972) as a generalization of a family of models on the dichotomous response level. Sufficient conditions that a model provides us with a unique modal point for the likelihood function of each and every response pattern have been investigated. In the homogeneous case, a sufficient condition is that, for an arbitrary item score x_g ($\neq 0$), the cumulative operating characteristic $P^*_{x_g}(\theta)$ is of Type A, i.e., strictly increasing in θ with zero and unity as its two asymptotes, and its asymptotic basic function, $\tilde{A}_{x_g}(\theta)$, which is defined by

$$\tilde{A}_{x_g}(\theta) = \frac{\partial}{\partial\theta}\left[\log\left\{\frac{\partial}{\partial\theta}P^*_{x_g}(\theta)\right\}\right] , \tag{5.1}$$

is strictly decreasing in θ. The satisfaction of this sufficient condition also implies two desirable features of the model such that: 1) the operating characteristic of each graded item score of each item has a single modal point, and 2) those modal points for a single item are

arranged in the same order as the item score itself. The normal ogive and logistic models, which have been generalized from the corresponding models on the dichotomous response level, are two examples of the models which satisfy the above sufficient condition.

These models of the homogeneous case on the graded response level can be generalized to provide us with those which belong to the general model of differential strategies. Let $\Psi(\theta)$ be a function of Type A. We shall consider the cumulative operating characteristic, $P^{*}_{y^{*}_{gsi}}(\theta)$, of the union of attainment categories y^{*}_{gsi} such that

$$P^{*}_{y^{*}_{gsi}}(\theta) = \beta_{y^{*}_{gsi}} \Psi(\theta - \alpha_{y^{*}_{gsi}}) \quad , \tag{5.2}$$

where $\alpha_{y^{*}_{g0i}}$ is negative infinity, $\alpha_{y^{*}_{g(m_{g}+1)i}}$ is positive infinity for $i = 1,2,\ldots,w$, and the values of $\alpha_{y^{*}_{gsi}}$ are ordered in the same way as those of s, for every strategy, and $\beta_{y^{*}_{gsi}}$ is a constant which satisfies

$$\sum_{j^{*}} \beta_{y^{*}_{gsj}} = \beta_{y^{*}_{g(s-1)i}} \quad , \tag{5.3}$$

with $\sum_{j^{*}}$ indicating the summation over all the strategies j branching from the point of the differentail strategy tree which is located right after the line representing the union $y^{*}_{g(s-1)i}$. From (5.3) it is obvious that, as far as there is no branching, $\beta_{y^{*}_{gsi}} = \beta_{y^{*}_{g(s-1)i}}$.

A sufficient condition that the model satisfies the unique maximum condition is: 1) that the values of the constant $\alpha_{y^{*}_{gsj}}$ are the same for

all the strategies j which branch from the vertex located immediately after the edge representing y^*_{gsi}, and 2) that we have

$$(5.4) \qquad \frac{\partial}{\partial\theta}\left[\log\{-\frac{\partial}{\partial\theta}\Psi(\theta)\}\right] < 0$$

almost everywhere in the domain of θ. To prove this, we obtain from (4.7), (5.2), (5.3) and the definition of the basic function $A_{y^*_{gsi}}(\theta)$, which was given by (4.10),

$$(5.5) \qquad A_{y^*_{gsi}}(\theta) = \frac{\partial}{\partial\theta}\log P_{y^*_{gsi}}(\theta)$$

$$= \frac{\partial}{\partial\theta}\log[\beta_{y^*_{gsi}}\ \Psi(\theta-\alpha_{y^*_{gsi}}) - \sum_{j^*}\beta_{y^*_{g(s+1)j}}\ \Psi(\theta-\alpha_{y^*_{g(s+1)j}})]$$

where $\sum_{j^*}$ indicates the summation over all the strategies j branching from the vertex which lies immediately after the line representing the union y^*_{gsi}. By virtue of the first condition, we can rewrite (5.5) in the form

$$(5.6) \qquad A_{y^*_{gsi}}(\theta) = \frac{\partial}{\partial\theta}\log\ [\beta_{y^*_{gsi}}\{\Psi(\theta-\alpha_{y^*_{gsi}}) - \ (\theta-\alpha_{y^*_{g(s+1)j}})\}]$$

$$= \frac{\partial}{\partial\theta}\log\ \{\Psi(\theta-\alpha_{y^*_{gsi}}) - \Psi(\theta-\alpha_{y^*_{g(s+1)j}})\}\ .$$

We notice that, if we replace y^*_{gsi} by the graded item score x_g and use $\Psi(\theta-\alpha_{x_g})$ as the cumulative operating characteristic $P^*_{x_g}(\theta)$, the last form of (5.6) is identical with the basic function of the graded item score, and the left hand side of (5.4) is identical with the corresponding asymptotic basic function. Thus we can say that all the unions, y^*_{gsi},

are equivalent to syndrome response categories (cf. Samejima, 1972, Section 5.2), and a unique maximum is assured for every possible response pattern.

If, for example, $\Psi(\theta)$ is a normal ogive function or a logistic distribution function, then (5.4) is satisfied (Samejima, 1972, Section 5.2), and we can develop the normal ogive model and the logistic model in the context of the general model for differential strategies, and both of them satisfy the unique maximum condition. In these two models, the cumulative operating characteristics are defined by

$$(5.7) \qquad P^{\star}_{y^{\star}_{gsi}}(\theta) = \beta_{y^{\star}_{gsi}}(2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta - b_{y^{\star}_{gsi}})} e^{-u^2/2}\, du$$

and

$$(5.8) \qquad P^{\star}_{y^{\star}_{gsi}}(\theta) = \beta_{y^{\star}_{gsi}}[1 + \exp\{-Da_g(\theta - b_{y^{\star}_{gsi}})\}]^{-1},$$

respectively, where a_g (>0) is the discrimination parameter specific for each problem g, $b_{y^{\star}_{gsi}}$ is a difficulty parameter defined for each union of attainment scores, with $b_{y^{\star}_{gOi}} = -\infty$ and $b_{y^{\star}_{g(m_{gi}+1)i}} = \infty$ and all those values are arranged in the same order as s with respect to each strategy, and D in (5.8) is a scaling factor which assumes 1.7 to retain the same set of parameter values as those in the normal ogive model.

For the purpose of illustration, we shall consider a differential strategy tree shown in Figure 5-1, which was taken from Example 1 given in

Section 1 and has only two successful strategies, or $w = 2$. Note, however, that this tree may represent either one of the digraphs shown in Figure 5-2. Figure 5-3 presents the set of ten cumulative operating characteristics $P^{*}_{y^{*}_{gsi}}(\theta)$, with $a_g = 1.00$, $b_{y^{*}_{g11}} = b_{y^{*}_{g12}} = -2.50$, $b_{y^{*}_{g21}} = -1.00$, $b_{y^{*}_{g31}} = 0.50$, $b_{y^{*}_{g22}} = -1.80$, $b_{y^{*}_{g32}} = 0.00$ and $b_{y^{*}_{g43}} = 2.00$, $\beta_{y^{*}_{g11}} = 0.60$ and $\beta_{y^{*}_{g12}} = 0.40$. The cumulative operating characteristic for y^{*}_{g01} is drawn by a solid line, and the three cumulative operating characteristics for the first strategy are drawn by dotted lines, the four for the second strategy by dashed lines, and the two for y^{*}_{g51} and y^{*}_{g62} coincide with the abscissa. Figure 5-4 presents the corresponding operating characteristics, in which all the modal points except for the negative and positive infinities are shown.

VI Single Strategy Case

Suppose that in solving the problem g there is only one successful strategy. Such a single strategy case can be considered as a special case of the multiple strategy case in which $w = 1$. The union of attainment scores, therefore, includes one and only one attainment score, and we can write

$$(6.1) \qquad y^{*}_{gsi} = (y_g = s) \ ,$$

where y_g is the attainment score of the unique successful strategy. Thus the three functions, i.e., the attainment function, the cumulative operating characteristic and the operating characteristic, are defined for each attainment score, rather than the union of attainment scores.

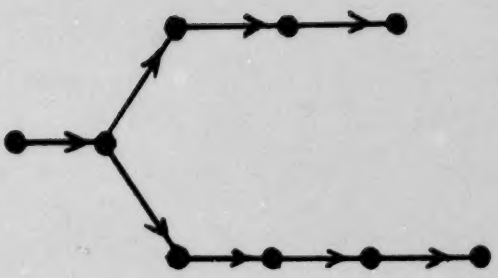

FIGURE 5-1

Differential Strategy Tree of Two Strategies Taken from Example 1.

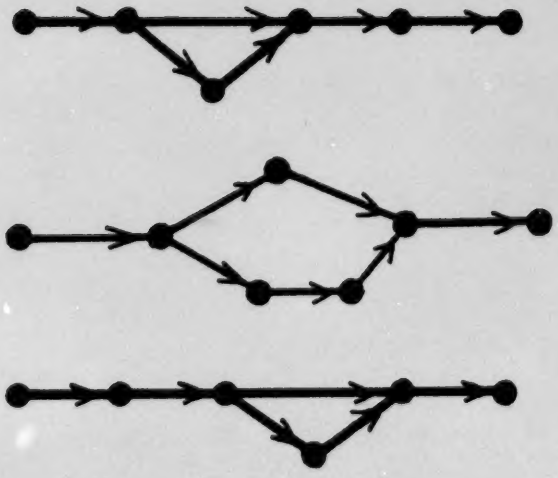

FIGURE 5-2

Three Digraphs Whose Differential Strategy Tree Is Given by Figure 5-1.

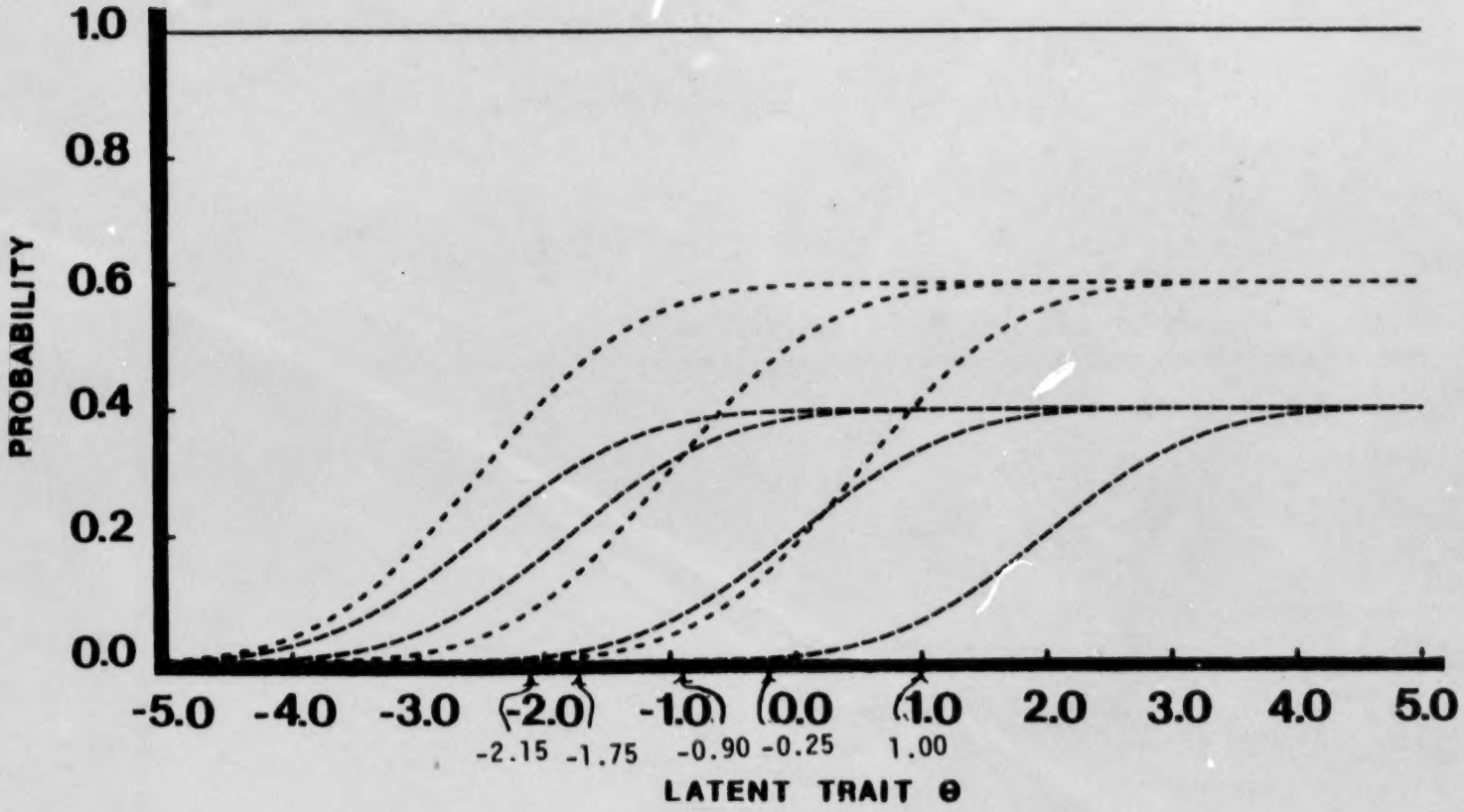

FIGURE 5-3

Cumulative Operating Characteristic of the Union of ($y_{g1} = 0$) and ($y_{g2} = 0$) (Solid Line), Those of ($y_{g1} = 1$), ($y_{g1} = 2$) and ($y_{g1} = 3$), Respectively (Dotted Line), and Those of ($y_{g2} = 1$), ($y_{g2} = 2$), ($y_{g2} = 3$) and ($y_{g2} = 4$), Respectively (Dashed Line).

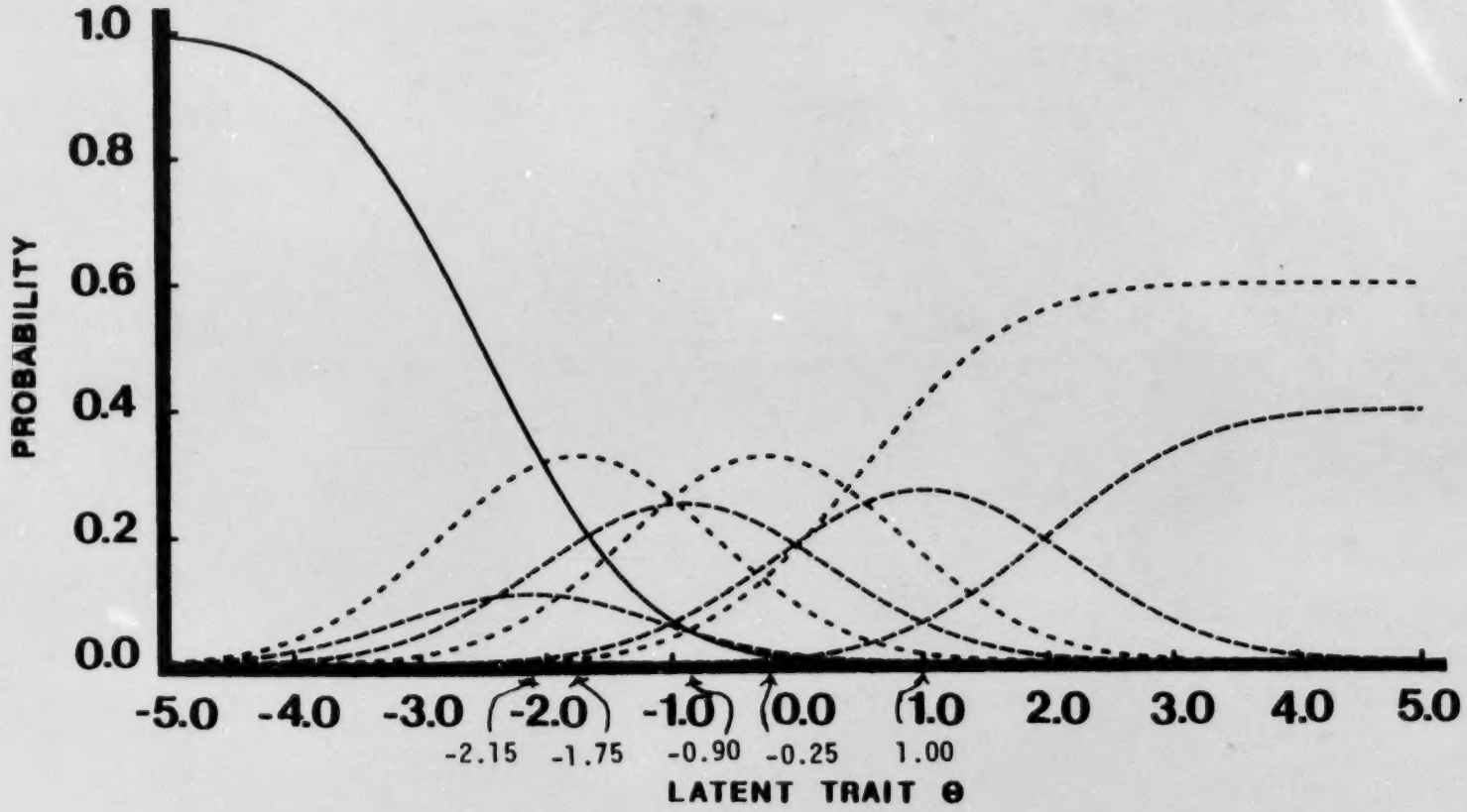

FIGURE 5-4

Operating Characteristic of the Union of ($y_{g1} = 0$) and ($y_{g2} = 0$) (Solid Line), Those of ($y_{g1} = 1$), ($y_{g1} = 2$) and ($y_{g1} = 3$), Respectively (Dotted Line), and Those of ($y_{g2} = 1$), ($y_{g2} = 2$), ($y_{g2} = 3$) and ($y_{g2} = 4$), Respectively (Dashed Line).

Let $M_{y_g}(\theta)$, $P^*_{y_g}(\theta)$ and $P_{y_g}(\theta)$ denote these three functions for the single strategy case. We can change the formulas (4.1), (4.2), (4.3), (4.7), (4.8) and (4.9) into much simplified forms, such that

$$(6.2) \qquad M_{y_g}(\theta) \begin{cases} = 1 & y_g = 0 \\ = 0 & y_g = m_g+1 \end{cases} ,$$

$$(6.3) \qquad P^*_{y_g}(\theta) = \prod_{j=0}^{y_g} M_j(\theta) \qquad y_g = 0,1,\ldots,m_g,(m_g+1) ,$$

$$(6.4) \qquad P^*_{y_g}(\theta) \begin{cases} = 1 & y_g = 0 \\ = M_{y_g}(\theta) & y_g = 1 \\ = 0 & y_g = m_g+1 \end{cases} ,$$

$$(6.5) \qquad P_{y_g}(\theta) = P^*_{y_g}(\theta) - P^*_{(y_g+1)}(\theta) \qquad y_g = 0,1,\ldots,m_g ,$$

$$(6.6) \qquad V = (y_1, y_2, \cdots, y_g, \cdots, y_n)'$$

and

$$(6.7) \qquad P_V(\theta) = \prod_{y_g \in V} P_{y_g}(\theta) ,$$

where m_g indicates the number of the subprocesses involved in solving the problem g.

We notice that these results are identical with those formulas for the graded response model (Samejima, 1972), with the replacement of the item score x_g by the attainment score y_g. Many findings concerning

the graded response model, therefore, can directly be applied for the single strategy case.

Let $A^*_{y_g}(\theta)$ be the first partial derivative of the natural logarithm of $P^*_{y_g}(\theta)$, such that

$$(6.8) \qquad A^*_{y_g}(\theta) = \frac{\partial}{\partial\theta} \log P^*_{y_g}(\theta) \qquad y_g = 0,1,\ldots,(m_g+1) .$$

It has been shown that the necessary and sufficient condition that $M_{y_g}(\theta)$ be strictly increasing in θ is that the inequality

$$(6.9) \qquad A^*_{y_g}(\theta) > A^*_{(y_g-1)}(\theta) \qquad y_g = 1,2,\ldots,(m_g+1)$$

holds almost everywhere with respect to θ (cf. Samejima, 1967, 1972).

In the homogeneous case $P^*_{y_g}(\theta)$ has zero and unity as its two asymptotes for $y_g = 1,2,\ldots,m_g$, and, furthermore, we can write

$$(6.10) \qquad P^*_s(\theta) = P^*_r(\theta - \alpha_{rs}) ,$$

where r and s are two arbitrarily selected attainment categories out of 1 through m_g with $r < s$, and α_{rs} is a positive finite constant. We obtain from (6.8) and (6.10)

$$(6.11) \qquad A^*_s(\theta) = A^*_r(\theta - \alpha_{rs}) .$$

From (6.9) and (6.11) it is obvious that a sufficient, though not necessary, condition that $M_{y_g}(\theta)$ be strictly increasing in θ for

$y_g = 1,2,\ldots,m_g$ is that $A^*_{y_g}(\theta)$ is strictly decreasing in θ for an arbitrarily chosen attainment category out of 1 through m_g. When m_g tends to positive infinity and α_{rs} for two adjacent real attainment categories tends to zero, this condition tends to the necessary and sufficient condition, for it requires that

$$(6.12) \qquad A^*_{y_g}(\theta) > A^*_{y_g}(\theta+\varepsilon)$$

for any small positive value of θ. Note that this condition is satisfied whenever the unique maximum condition is satisfied. Above all, when the asymptotic basic function, $\tilde{A}^*_{y_g}(\theta)$, which is defined by

$$(6.13) \qquad \tilde{A}^*_{y_g}(\theta) = \left[\frac{\partial^2}{\partial\theta^2} P^*_{y_g}(\theta)\right]\left[\frac{\partial}{\partial\theta} P^*_{y_g}(\theta)\right]^{-1} ,$$

is strictly decreasing in θ, not only $M_{y_g}(\theta)$ of each subprocess is strictly increasing in θ, however finely differentiated it may be, but also a unique maximum is assured for the likelihood function of each and every possible response pattern which consists of such attainment scores of different tasks (cf. Samejima, 1972, Sections 5.1 and 5.2). It has been shown (Samejima, 1967, 1972) that in the normal ogive model and in the logistic model on the graded response level (Samejima, 1969), for example, this condition is satisfied. In the former example, we can write

$$(6.14) \qquad P^*_{y_g}(\theta) = (2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta-b_{y_g})} \exp(-u^2/2)\, du$$

and in the latter

$$(6.15) \qquad P^{*}_{y_g}(\theta) = [1 + \exp\{-Da_g(\theta - b_{y_g})\}]^{-1} \quad ,$$

where a_g is the item discrimination parameter and b_{y_g} is the difficulty parameter for each attainment category y_g, and D is a scaling factor which is usually set equal to 1.7 (Birnbaum, 1968). In both models, the upper asymptote of $M_{y_g}(\theta)$ for $y_g = 1,2,\ldots,m_g$ is unity, while the lower asymptote is zero in the normal ogive model and $\exp[-Da_g(b_{y_g} - b_{(y_g-1)})]$ in the logistic model. This lower asymptote in the logistic model depends upon the distance between the difficulty parameters of the two adjacent attainment categories, assuming zero for $y_g = 1$ and positive numbers less than unity otherwise. In both models, $M_{y_g}(\theta)$ for $y_g = 2,3,\ldots,m_g$ tends to unity for the entire range of θ as b_{y_g} approaches $b_{(y_g-1)}$, and tends to $P^{*}_{y_g}(\theta)$ as b_{y_g} departs from $b_{(y_g-1)}$ (cf. Samejima, 1972, Figure 5-2-1).

We shall consider an example of the problem solving task, in which we can assume a single successful strategy. The problem was selected from the thirty paper-and-pencil test items of the LIS Measurement Scale for Non-Verbal Reasoning Ability (Indow and Samejima, 1962, 1966). The test item is called $D_{2.2}$, and is shown in Figure 6-1. There are three Japanese words written by English alphabetical letters, "YAMA," "UMI," and "KAWA," which consist of one, and only one, of the seven one-digit numbers, which are shown in the brackets in Figure 6-1. The sum of the

first two words is supposed to equal the third word, and the examinee is expected to find out which letter represents which number.

In solving this problem, we notice that: 1) "I" is added to "A" to produce "A" itself with no carried number added, so "I" must equal 0 ; 2) a similar result is observed for "A" and "U", but 0 is already assigned to "I", so "U" must be 9 ; 3) "K" must be greater than "Y" by one, so (Y,K) must be either one of (3,4) , (6,7) and (7,8) ; 4) since 1 must be carried from the second lowest digit to the third, two times "M" must be "W" plus ten, so (M,W) must be either (7,4) or (8,6) ; 5) combining 3) and 4), since no overlappings are allowed, we we must conclude (Y,K) = (3,4) and (M,W) = (8,6); 6) since 7 is the only remaining number, "A" must be 7 . Note that we could reverse 1) 2) 3) to 3) 2) 1) , since 3) can be reached independently and it gives a sufficient information to discover 2) . Also 4) does not need both 1) and 3) , since we can reach it directly from 2) . For these reasons, strictly speaking, there are some minor variations in this strategy, and the above problem is not exactly an example of the single strategy case. In spite of those variations, however, our observation tells us that the majority of subjects take the above strategy, probably because 1) is easier to discover than 3), and so forth. Table 6-1 presents the attainment scores and the description of the corresponding subprocess for each score in this example.

It has been reported (Samejima, 1969) that, in their effort of solving this problem in the paper-and-pencil testing situation, as many as 187 subjects out of the total of 883 junior high school students

```
  Y A M A          (0 3 4 6 7 8 9)
+ U M I            A I K M U W Y
-------            || || || || || || ||
K A W A
```

FIGURE 6-1

Item $D_{2.2}$ of LIS Measurement Scale of Non-Verbal Ability.

TABLE 6-1

Subprocesses of the Task Involved in Solving Item $D_{2.2}$ of LIS Measurement Scale of Non-Verbal Ability.

Item Score x_g	Attainment Score y_g	Subprocess
0	0	(none)
0	1	I = 0 Left: (3 4 6 7 8 9)
1	2	U = 9 Left: (3 4 6 7 8)
1	3	K = Y + 1 (Y,K) = (3,4) or (6,7) or (7,8)
1	4	W = 2M - 10 (M,W) = (7,4) or (8,6)
1	5	M = 8 W = 6 Y = 3 K = 4 Left: (7)
2	6	A = 7

discovered both $I = 0$ and $U = 9$ and wrote them down on their sheets, although they did not reach the complete solution of the problem. The number of those subjects is almost as large as that of those who reached the complete answer, i.e., 202 . For this reason, using the item score x_g $(= 0,1,2)$, which is shown in the first column of Table 6-1, the operating characteristic has been estimated for each item score using the normal ogive model on the graded response level. The resultant three operating characteristics, $P_{x_g}(\theta)$ for $x_g = 0,1,2$, have been reported, and are reproduced as Appendix. It has also been reported that this result provides us with a good internal consistency which supports the validity of the model.

VII Discussion and Conclusions

Using digraphs and trees, an attempt was made to give a rationale behind a general model for differential strategies, and some concrete models in the homogeneous case were proposed. The single strategy case was observed as a special case of the multiple strategy case.

A question may arise as to which estimate of the latent trait should be taken if the subject faltered from one strategy to another and did not reach the solution of the problem. One answer to this question may be to take the attainment score of the strategy that he took last, and use its corresponding operating characteristic in estimating his latent trait. An alternative answer may be to compare the resultant estimates of θ obtained by choosing the different strategies the subject has taken and select the highest estimate.

The usefulness of the model is yet to be discovered. The example of the item $D_{2.2}$ of the LIS Measurement Scale provides us, however, with some support for considering cognitive processes in the framework of latent trait theory.

One of the difficulties in any quantification of cognitive data may be their small sample sizes. It will be useful to conduct intensive, observant laboratory studies as a pilot study, and then to use their results in designing more extensive studies, like paper-and-pencil testing, computerized adaptive testing, and so forth, for the purpose of estimating the operating characteristics and eventually the subject's latent trait.

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APPENDIX

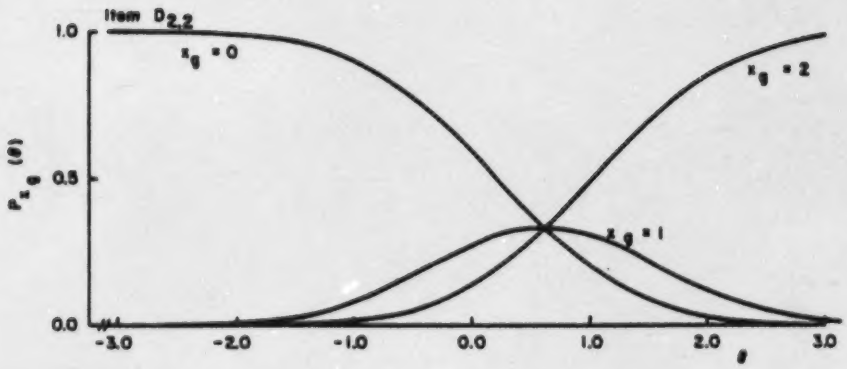

FIGURE 5-1

Operating characteristics of graded responses for items C_1 and $D_{2.2}$ of LIS measurement scale for Non-verbal Reasoning Ability on the normal ogive model.

Navy

1 Dr. Ed Aiken
Navy Personnel R&D Center
San Diego, CA 92152

1 CDR Robert J. Biersner
Naval Medical R&D Command
National Naval Medical Center
Bethesda, MD 20814

1 Liaison Scientist
Office of Naval Research
Branch Office, London
Box 39
FPO New York, NY 09510

1 Lt. Alexander Bory
Applied Psychology
Measurement Division
NAMRL
NAS Pensacola, FL 32508

1 Dr. Robert Carroll
NAVOP 115
Washington , DC 20370

1 Dr. Stanley Collyer
Office of Naval Technology
800 N. Quincy Street
Arlington, VA 22217

1 DR. PAT FEDERICO
Code P13
NPRDC
San Diego, CA 92152

1 Dr. John Ford
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Norman J. Kerr
Chief of Naval Technical Training
Naval Air Station Memphis (75)
Millington, TN 38054

1 Dr. William L. Maloy (02)
Chief of Naval Education and Training
Naval Air Station

Pensacola, FL 32508

1 Dr. Kneale Marshall
Chairman, Operations Research Dept.
Naval Post Graduate School
Monterey, CA 93940

Navy

1 Dr. James McBride
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr William Montague
NPRDC Code 13
San Diego, CA 92152

1 Bill Nordbrock
1032 Fairlawn Ave.
Libertyville, IL 60048

1 Library, Code P201L
Navy Personnel R&D Center
San Diego, CA 92152

1 Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

6 Commanding Officer
Naval Research Laboratory
Code 2627
Washington, DC 20390

6 Personnel & Training Research Group
Code 442PT
Office of Naval Research
Arlington, VA 22217

1 LT Frank C. Petho, MSC, USN (Ph.D)
CNET (N-432)
NAS
Pensacola, FL 32508

1 Dr. Bernard Rimland (01C)
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

1 Dr. Worth Scanland
CNET (N-5)
NAS, Pensacola, FL 32508

1 Dr. Robert G. Smith
Office of Chief of Naval Operations
OP-987H
Washington, DC 20350

Navy

1 Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Frederick Steinheiser
CNO - OP115
Navy Annex
Arlington, VA 20370

1 Mr. Brad Sympson
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Ronald Weitzman
Code 54 WZ
Department of Administrative Sciences
U. S. Naval Postgraduate School
Monterey, CA 93940

1 Dr. Douglas Wetzel
Code 12
Navy Personnel R&D Center
San Diego, CA 92152

1 DR. MARTIN F. WISKOFF
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

1 Mr John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152

Marine Corps

1 H. William Greenup
Education Advisor (E031)
Education Center, MCDEC
Quantico, VA 22134

1 Director, Office of Manpower Utilizatio
HQ, Marine Corps (MPU)
BCB, Bldg. 2009
Quantico, VA 22134

1 Headquarters, U. S. Marine Corps
Code MPI-20
Washington, DC 20380

1 Special Assistant for Marine
Corps Matters
Code 100M
Office of Naval Research
800 N. Quincy St.
Arlington, VA 22217

1 DR. A.L. SLAFKOSKY
SCIENTIFIC ADVISOR (CODE RD-1)
HQ, U.S. MARINE CORPS
WASHINGTON, DC 20380

1 Major Frank Yohannan, USMC
Headquarters, Marine Corps
(Code MPI-20)
Washington, DC 20380

Army

1 Technical Director
U. S. Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Blvd.
Alexandria , VA 22333

1 Dr. Myron Fischl
U.S. Army Research Institute for the
Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Milton S. Katz
Training Technical Area
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Mr. Robert Ross
U.S. Army Research Institute for the
Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Hilda Wing
Army Research Institute
5001 Eisenhower Ave.
Alexandria, VA 22333

1 Dr. Robert Wisher
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Air Force

1 Technical Documents Center
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WPAFB, OH 45433

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Research
Life Sciences Directorate, NL
Bolling Air Force Base
Washington, DC 20332

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HQ, AFHRL (AFSC)
Brooks AFB, TX 78235

1 Mr. Raymond E. Christal
AFHRL/MOE
Brooks AFB, TX 78235

1 Dr. Alfred R. Fregly
AFOSR/NL
Bolling AFB, DC 20332

1 Dr. Roger Pennell
Air Force Human Resources Laboratory
Lowry AFB, CO 80230

1 Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

Department of Defense

12 Defense Technical Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC

1 Jerry Lehnus
HQ MEPCOM
Attn: MEPCT-P
Fort Sheridan, IL 60037

1 Military Assistant for Training and
Personnel Technology
Office of the Under Secretary of Defens
for Research & Engineering
Room 3D129, The Pentagon
Washington, DC 20301

1 Dr. Wayne Sellman
Office of the Assistant Secretary
of Defense (MRA & L)
2B269 The Pentagon
Washington, DC 20301

1 Major Jack Thorpe
DARPA
1400 Wilson Blvd.
Arlington, VA 22209

Civilian Agencies

1 Dr. Susan Chipman
Learning and Development
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E Street NW
Washington, DC 20415

1 Mr. Thomas A. Warm
U. S. Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

1 Dr. Frank Withrow
U. S. Office of Education
400 Maryland Ave. SW
Washington, DC 20202

1 Dr. Joseph L. Young, Director
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Private Sector

1 Dr. James Algina
University of Florida
Gainesville, FL 326

1 Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

1 Psychological Research Unit
Dept. of Defense (Army Office)
Campbell Park Offices
Canberra ACT 2600
AUSTRALIA

1 Dr. Alan Baddeley
Medical Research Council
Applied Psychology Unit
15 Chaucer Road
Cambridge CB2 2EF
ENGLAND

1 Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

1 Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Tel Aviv, Ramat Aviv 69978
Israel

1 Dr. R. Darrel Bock
Department of Education
University of Chicago
Chicago, IL 60637

1 Dr. Robert Brennan
American College Testing Programs
P. O. Box 168
Iowa City, IA 52243

1 Bundministerium der Verteidigung
-Referat P II 4-
Psychological Service
Postfach 1328
D-5300 Bonn 1
F. R. of Germany

1 Dr. Ernest R. Cadotte
307 Stokely
University of Tennessee
Knoxville, TN 37916

Private Sector

1 Dr. Pat Carpenter
Department of Psychology
Carnegie-Mellon University
Pittsburgh, PA 15213

1 Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

1 Dr. Norman Cliff
Dept. of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

1 Dr. Allan M. Collins
Bolt Beranek & Newman, Inc.
50 Moulton Street
Cambridge, MA 02138

1 Dr. Lynn A. Cooper
LRDC
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15213

1 Dr. Fritz Drasgow
Department of Psychology
University of Illinois
603 E. Daniel St.
Champaign, IL 61820

1 Dr. Susan Embertson
PSYCHOLOGY DEPARTMENT
UNIVERSITY OF KANSAS
Lawrence, KS 66045

1 ERIC Facility-Acquisitions
4833 Rugby Avenue
Bethesda, MD 20014

1 Dr. Benjamin A. Fairbank, Jr.
McFann-Gray & Associates, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

1 Dr. Leonard Feldt
Lindquist Center for Measurment
University of Iowa
Iowa City, IA 52242

Private Sector

1 Dr. Richard L. Ferguson
The American College Testing Program
P.O. Box 168
Iowa City, IA 52240

1 Univ. Prof. Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

1 Professor Donald Fitzgerald
University of New England
Armidale, New South Wales 2351
AUSTRALIA

1 Dr. Dexter Fletcher
WICAT Research Institute
1875 S. State St.
Orem, UT 22333

1 Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138

1 Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01002

1 Dr. Robert Glaser
Learning Research & Development Center
University of Pittsburgh
3939 O'Hara Street
PITTSBURGH, PA 15260

1 Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

1 Dr. Ron Hambleton
School of Education
University of Massachusetts
Amherst, MA 01002

1 Dr. Lloyd Humphreys
Department of Psychology
University of Illinois
Champaign, IL 61820

Private Sector

1 Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

1 Dr. Earl Hunt
Dept. of Psychology
University of Washington
Seattle, WA 98105

1 Dr. Huynh Huynh
College of Education
University of South Carolina
Columbia, SC 29208

1 Dr. Douglas H. Jones
Room T-255/21-T
Educational Testing Service
Princeton, NJ 08541

1 Professor John A. Keats
University of Newcastle
N. S. W. 2308
AUSTRALIA

1 Mr. Jeff Kelety
Department of Instructional Technology
University of Southern California
Los Angeles, CA 92007

1 Dr. Scott Kelso
Haskins Laboratories, Inc
270 Crown Street
New Haven, CT 06510

1 Dr. Stephen Kosslyn
Department of Psychology
The Johns Hopkins University
Baltimore, MD 21218

1 Dr. Alan Lesgold
Learning R&D Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

1 Dr. Michael Levine
Department of Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Private Sector

1 Dr. Charles Lewis
Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Oude Boteringestraat 23
9712GC Groningen
Netherlands

1 Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

1 Mr. Phillip Livingston
Systems and Applied Sciences Corporatio
6811 Kenilworth Avenue
Riverdale, MD 20840

1 Dr. Robert Lockman
Center for Naval Analysis
200 North Beauregard St.
Alexandria, VA 22311

1 Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

1 Dr. James Lumsden
Department of Psychology
University of Western Australia
Nedlands W.A. 6009
AUSTRALIA

1 Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

1 Dr. Scott Maxwell
Department of Psychology
University of Notre Dame
Notre Dame, IN 46556

1 Dr. Samuel T. Mayo
Loyola University of Chicago
820 North Michigan Avenue
Chicago, IL 60611

1 Mr. Robert McKinley
American College Testing Programs
P.O. Box 168
Iowa City, IA 52243

Private Sector

1
Professor Jason Millman
Department of Education
Stone Hall
Cornell University
Ithaca, NY 14853

1 Dr. Allen Munro
Behavioral Technology Laboratories
1845 Elena Ave., Fourth Floor
Redondo Beach, CA 90277

1 Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

1 Dr. Donald A Norman
Cognitive Science, C-015
Univ. of California, San Diego
La Jolla, CA 92093

1 Dr. Melvin R. Novick
356 Lindquist Center for Measurment
University of Iowa
Iowa City, IA 52242

1 Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

1 Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

1 Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Cirle, NW
Washington, DC 20036

1 Dr. James A. Paulson
Portland State University
P.O. Box 751
Portland, OR 97207

1 Dr. James W. Pellegrino
University of California,
Santa Barbara
Dept. of Psychology
Santa Barabara, CA 93106

Private Sector

1 Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

1 Dr. Thomas Reynolds
University of Texas-Dallas
Marketing Department
P. O. Box 688
Richardson, TX 75080

1 Dr. Andrew M. Rose
American Institutes for Research
1055 Thomas Jefferson St. NW
Washington, DC 20007

1 Dr. Ernst Z. Rothkopf
Bell Laboratories
Murray Hill, NJ 07974

1 Dr. David Rumelhart
Center for Human Information Processing
Univ. of California, San Diego
La Jolla, CA 92093

1 Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

1 PROF. FUMIKO SAMEJIMA
DEPT. OF PSYCHOLOGY
UNIVERSITY OF TENNESSEE
KNOXVILLE, TN 37916

1 Dr. Walter Schneider
Psychology Department
603 E. Daniel
Champaign, IL 61820

1 Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

1 Dr. Kazuo Shigemasu
University of Tohoku
Department of Educational Psychology
Kawauchi, Sendai 980
JAPAN

Private Sector

1 Dr. Edwin Shirkey
Department of Psychology
University of Central Florida
Orlando, FL 32816

1 Dr. William Sims
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

1 Dr. Richard Snow
School of Education
Stanford University
Stanford, CA 94305

1 Dr. Elliott Soloway
Yale University
Department of Computer Science
P.O. Box 2158
New Haven, CT 06520

1 Dr. Robert Sternberg
Dept. of Psychology
Yale University
Box 11A, Yale Station
New Haven, CT 06520

1 Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

1 David E. Stone, Ph.D.
Hazeltine Corporation
7680 Old Springhouse Road
McLean, VA 22102

1 Dr. William Stout
University of Illinois
Department of Mathematics
Urbana, IL 61801

1 DR. PATRICK SUPPES
INSTITUTE FOR MATHEMATICAL STUDIES IN
THE SOCIAL SCIENCES
STANFORD UNIVERSITY
STANFORD, CA 94305

1 Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

62

Private Sector

1 Dr. Kikumi Tatsuoka
Computer Based Education Research Lab
252 Engineering Research Laboratory
Urbana, IL 61801

1 Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

1 Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

1 Dr. Douglas Towne
Univ. of So. California
Behavioral Technology Labs
1845 S. Elena Ave.
Redondo Beach, CA 90277

1 Dr. Robert Tsutakawa
Department of Statistics
University of Missouri
Columbia, MO 65201

1 Dr. V. R. R. Uppuluri
Union Carbide Corporation
Nuclear Division
P. O. Box Y
Oak Ridge, TN 37830

1 Dr. David Vale
Assessment Systems Corporation
2233 University Avenue
Suite 310
St. Paul, MN 55114

1 Dr. Kurt Van Lehn
Zerox PARC
3333 Coyote Hill Road
Palo Alto, CA 94304

1 Dr. Howard Wainer
Division of Psychological Studies
Educational Testing Service
Princeton, NJ 08540

1 Dr. Brian Waters
HumRRO
300 North Washington
Alexandria, VA 22314

Private Sector

1 Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

1 Dr. Christopher Wickens
Department of Psychology
University of Illinois
Champaign, IL 61820

1 Dr. Rand R. Wilcox
University of Southern California
Department of Psychology
Los Angeles, CA 90007

1 Wolfgang Wildgrube
Streitkraeftesmt
Box 20 50 03
D-5300 Bonn 2
WEST GERMANY

1 Dr. Bruce Williams
Department of Educational Psychology
University of Illinois
Urbana, IL 61801

1 Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

-49-

63

Navy

1 Ms. Nancy McMan
Office of Naval Research
206 O'Keefe Building
Atlanta, GA 30332

Army

1 Dr. Randall M. Chambers
U.S. Army Research Institute for
the Behavioral & Social Sciences
Fort Sill Field Unit
P.O. Box 3066
Fort Sill, OK 73503

Non Govt

1 Dr. Robert Guion
Department of Psychology
Bowling Green State University
Bowling Green, OH 43403

1 Dr. Niel Tinn
Department of Information Systems
Planning
University of Pittsburgh
Pittsburgh, PA 15260

1 Dr. William W. Turnbull
Educational Testing Service
Princeton, NJ 08542

1 Dr. P. Mengal
Faculte' de Psychologie
et des Sciences de l'Education
Universite' de Geneve
3 fl. de l'Universite
1201 Geneva SWITZERLAND

1 Dr. Wim J. van der Linden
Vakgroep Onderwisjskunde
Postbus 217
7500 EA Enschede
THE NETHERLANDS

1 Dr. Lutz Hornke
University Duesseldorf
Erz. Wiss.
D-4000 Duesseldorf
WEST GERMANY

1 Dr. Wolfgang Buchtala
8346 Simbach Inn
Postfach 1306
Industriestrasse 1
WEST GERMANY

1 Dr. Albert Beaton
Educational Testing Service
Princeton, NJ 08542

1 Dr. Sukeyori Shiba
Faculty of Education
University of Tokyo
Hongo, Bunkyoku
Tokyo, JAPAN 113

Non Govt

1 Dr. Yukihiro Noguchi
1208-7 Kamoi, Midoriku
Yokohama 216, JAPAN

1 Dr. Takahiro Sato
Nippon Electric Co., Ltd.
C & C Systems Research Laboratories
4-1-1 Miyazaki
Miyamaeku, Kawasaki
Kanagawaken 213, JAPAN

1 Dr. James Chan
354 Lindquist Center for Measurement
University of Iowa
Iowa City, Iowa 52240

1 Dr. Douglas Carroll
Bell Laboratories
600 Mountain Ave.
Murray Hill, N.J. 07974

1 Dr. Lowell Schipper
Department of Psychology
Bowling Green State University
Bowling Green, OH 43403

1 Dr. J. Uhlaner
Perceptronics, Inc.
6271 Variel Avenue
Woodland Hills, CA 91634

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